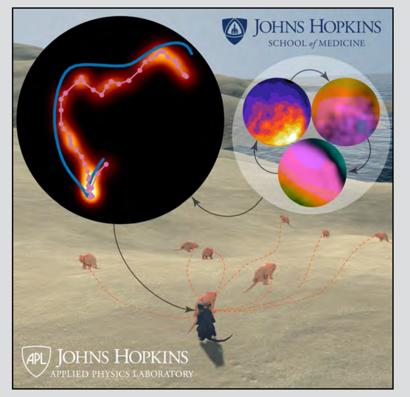

Self-organized swarm control using neural principles of spatial phase coding

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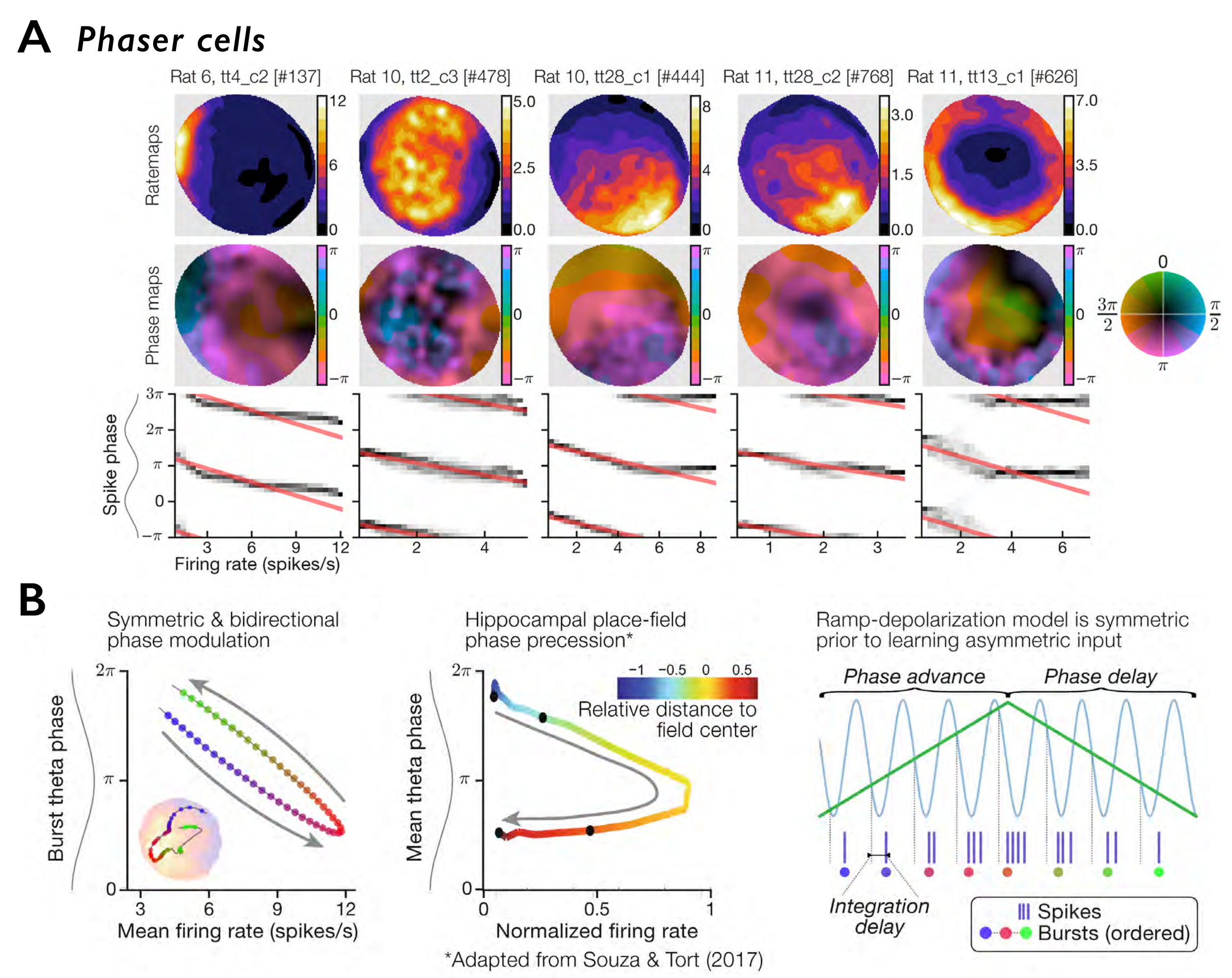
NCS/FO Spatial Intelligence for Swarms Based on Hippocampal Dynamics

The ubiquity of small multi-agent robotic platforms may help proliferate applications of autonomous technologies, but current control algorithms lack robustness and computational efficiency in complex environments. We suggest that this gap reflects the need for effective future-oriented reasoning for online replanning of distributed spatial tasks. Crucially, solutions to prospective spatial reasoning have evolved biologically in animals. Here, we introduce our analogy between the rodent brain circuits for spatial navigation and artificial swarms in the context of a recently discovered neuron, phaser cells, whose spatial activity we characterized as a strong, symmetric coupling between firing rate and spike phase relative to hippocampal theta oscillations. Phaser cells, found largely in the lateral septum and other subcortical regions (i.e., hippocampus, anteroventral thalamus, lateral hypothalamus, and nucleus accumbens), encoded space by assigning distinct phases to isocontour levels of external spatial inputs. In simulations, competitive learning embedded spatial phase codes into the weights of downstream path-integration networks and Bayesian decoding analysis revealed error correction at subsecond timescales. Thus, we outline a neurocomputational strategy in which location-dependent synchrony reconciles internal self-motion with external reference points and extend this strategy to the problem of online control of artificial robotic swarms. Our analogy suggests that spatial neurons and their firing fields are mathematically equivalent to robotic agents and their current positions. We assigned an internal phase state to each agent that is communicated to local neighbors using low-bandwidth pulsatile communication akin to phasic spiking. We show how the coupled spatial and phase dynamics of swarmalators with allothetic sensory inputs recapitulate the flexible spatial synchronization patterns observed in phaser cell simulations. Our findings demonstrate that swarming tasks including target tracking, patrol, and obstacle avoidance can benefit from this neural control paradigm to qualitatively improve agility, efficiency, and robustness in realistic environments.

Phaser cells & Swarmalators



C Swarmalators

Phase-Coupled Attraction

$$\dot{\mathbf{x}}_i = \frac{1}{N}\left[\sum_{j\notin i}^{N} \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|}\left(A + J\cos(\theta_j - \theta_i)\right) - B\frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|^2}\right]$$

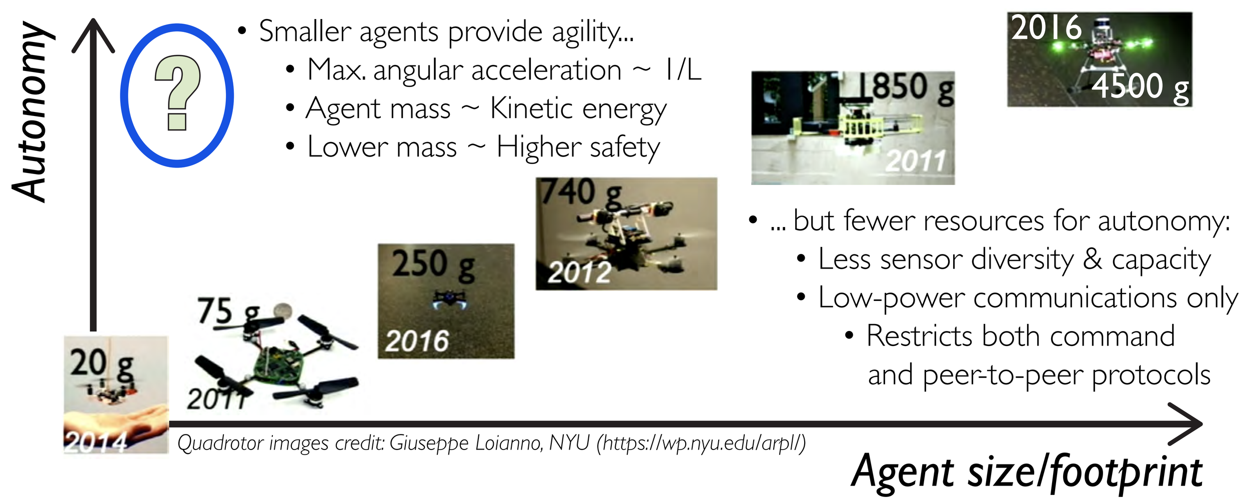
Local Kuramoto Phase Synchronization

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j\notin i}^{N}\frac{\sin(\theta_j - \theta_i)}{|\mathbf{x}_j - \mathbf{x}_i|}$$

O'Keeffe, Hong, & Strogatz. (2017). "Oscillators that sync and swarm." Nat Comm, 8(1), 1504.

Fig. 1. Two characterizations from recent research in neuroscience and applied mathematics motivated the ideas and approach of the current project. (A) Phaser cells are theta-rhythmic neurons located predominantly in the lateral septum. They were characterized by Monaco et al. (2019), supported by this NCS/FO award, as carrying a strong firing rate-coupled phase code for space. (B) The phaser cell code was theorized as a symmetric coupling distinct from typical hippocampal phase precession that may serve computational roles in path integration or other functions of spatial navigation. (C) Swarmalators were developed by O'Keeffe, Hong, & Strogatz (2017) as a mathematical dynamical system that generalizes a class of collective biological behaviors observed in groups of animals, insects, and bacteria. The key idea is that mobile entities have an intrinsic phase state that that synchronizes (K>0) or desynchronizes (K<0) the population via Kuramoto coupling and guides the movements of individual agents. Groups of these 'mobile oscillators' can exhibit robust self-organizing dynamics.

Current approaches to autonomous control of multi-agent groups of robotic vehicles



- ***High-dimensional state estimation via sensor fusion (camera, IMU, etc.)***

$$\boldsymbol{x} = \left[\ \boldsymbol{x}_{sb}^\top\ \boldsymbol{\Theta}_{sb}^\top\ \boldsymbol{v}_{sb}^\top\ \boldsymbol{\gamma}^\top\ \boldsymbol{b}_g^\top\ \boldsymbol{b}_a^\top\ \right]^\top$$

... but the sensor model is dependent on both the state and the input:

$$\boldsymbol{z}_{\text{camera}} = \boldsymbol{f}_{\text{camera}}\left(\boldsymbol{x}(t), \boldsymbol{u}(t)\right) + \eta_{\text{camera}} \qquad \boldsymbol{z}_{\text{IMU}} = \boldsymbol{f}_{\text{IMU}}\left(\boldsymbol{x}(t), \boldsymbol{u}(t)\right) + \eta_{\text{IMU}}$$

- ***Critical applications require agility + useful sensory model estimates in complex, changing environments that make perception difficult:***
  - Damaged or unknown infrastructure
  - Confined spaces
  - GPS-denied areas
  - Variable lighting or darkness
  - Precipitation, moisture, or fog
  - Small or complex access points
- ***Larger groups of smaller agents (increasing N, decreasing mass) may provide a way, but current goal-directed control methods do not scale,***

$$\boldsymbol{X}(t) = \left[\ \boldsymbol{x_1}(t)^\top \dots\ \boldsymbol{x_N}(t)^\top\ \right]^\top \qquad \boldsymbol{G} = \left[\ \boldsymbol{g_1}(t)^\top \dots\ \boldsymbol{g_M}(t)^\top\ \right]^\top$$

... because they rely on exhaustive search and/or optimization procedures:

$$\min \int_{t_0}^{t} \left|\left|\frac{d\boldsymbol{X}(t)}{dt}\right|\right|^2 dt \qquad \boldsymbol{\Phi}^* = \arg\min \sum_{i=1}^{N}\sum_{j=1}^{M} \boldsymbol{\Phi}_{i,j}||\boldsymbol{x}_i(t) - \boldsymbol{g}_j||^2$$

Chung et al. (2018); Weinstein, Cho, Loianno, & Kumar (2018)

'Stiefelators' for generalizing phase variables

- ***The Stiefel manifold $V_1(\mathbb{R}^2)$ defines the unit circle in the plane. This space is naturally identified with angles θ by $X(\theta_i) = (\cos(\theta), \sin(\theta))^\top$, such that for two angles θ_i and θ_j, it follows that $\cos(\theta_j - \theta_i) = \langle X(\theta_i), X(\theta_j)\rangle$.***
- ***The Kuramoto synchronization of swarmalators can be cast in geometric terms to be extended to arbitrary Stiefel manifolds [5] and product spaces of Stiefel manifolds [6]. This suggests a basic form of generalization of swarmalator dynamics to Stiefel manifolds as***

$$\dot{\boldsymbol{x}} = \frac{1}{N}\sum_{j=1}^{N}\left[I_{att}(\boldsymbol{x}_j - \boldsymbol{x}_i)\langle X_j, X_i\rangle - I_{rep}(\boldsymbol{x}_j - \boldsymbol{x}_i)\right]$$

$$\dot{X}_i = X_i\Omega_i + \frac{K}{N}\left(I - X_iX_i^\dagger\right)\sum_{j=1}^{N} X_j G(\boldsymbol{x}_j - \boldsymbol{x}_i)$$

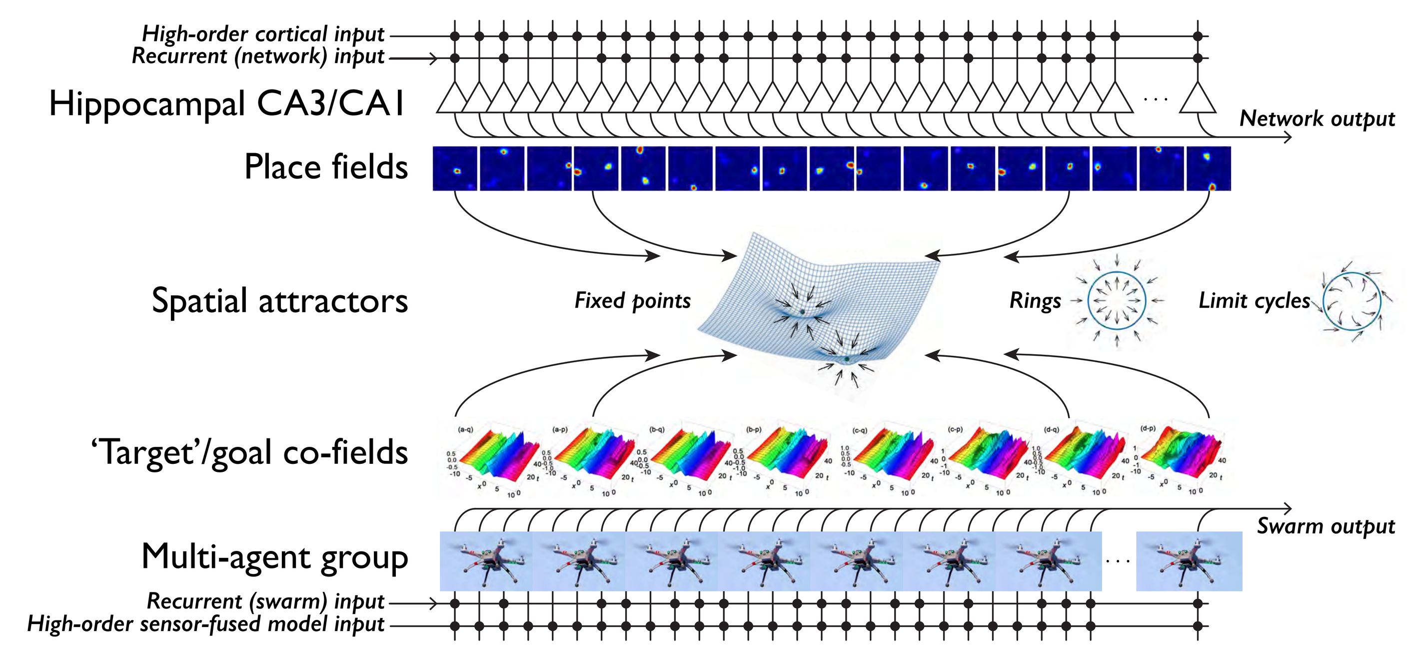
where X_i is the auxiliary Stiefel manifold state and Ω_i is a skew-symmetric matrix of appropriate dimension.

- ***Since angles can be viewed through the lens of Stiefel manifolds, we consider this an implicit representation of neural network models when an angular variable is converted to a unit vector inside the computations. This applies to models such as continuous attractor grid-cell networks [7] and oscillatory interference for path integration [8].***

Swarmalator aggregation with Kilobots

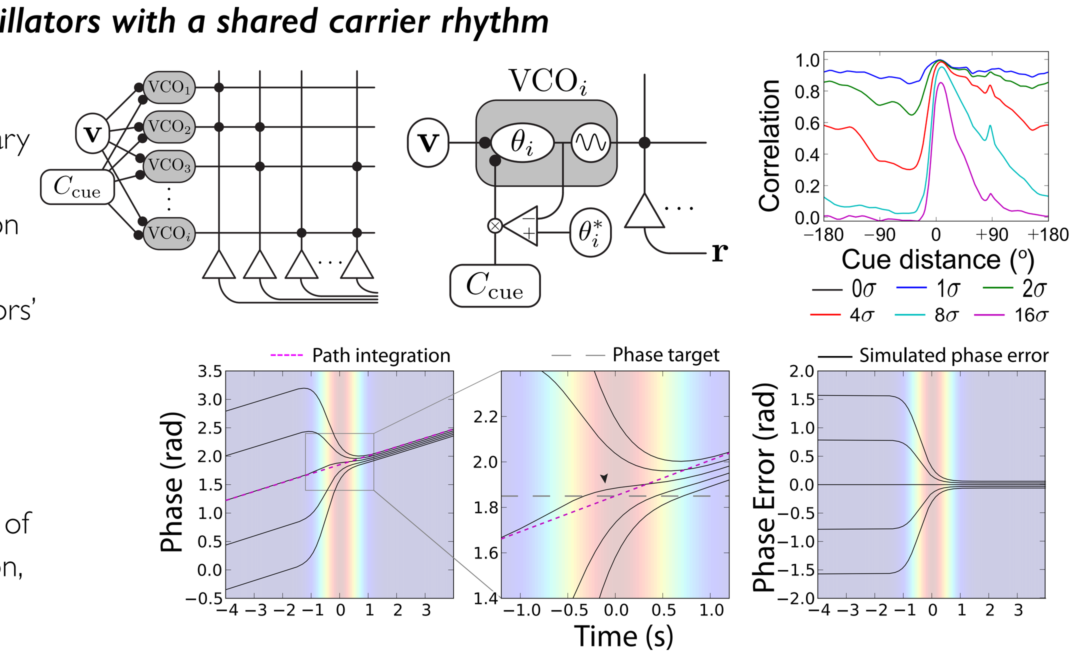
Fig. 2. A small swarm of kilobots that has implemented basic swarmalator control using color LED transceivers that detect phase information reflected off the surface. We have demonstrated simple swarming behaviors such as self-organized aggregation, but equilibrium-based control methods are fundamentally limited in more complex problems. Video credit: Bryanna Yeh, JHU/APL.vv

The agent–neuron analogy as the key to neural self-organized autonomous control



Monaco & Abbott (2011); Knierim & Zhang (2012); Ivancevic & Reid (2016)

- ***Landmark-based feedback can drive decentralized correction of path integration among motion-dependent oscillators with a shared carrier rhythm***
  - Oscillatory interference theory suggests that arbitrary spatial functions can be encoded by the organization of phase-coding neurons: 'velocity-controlled oscillators' (VCOs; Blair et al., 2010).
  - Cue-driven feedback, perhaps via phaser cells (Monaco et al., 2019), provides robust correction of self-motion-based navigation, such as path integration.



Monaco, Knierim, & Zhang (2011)

Spatial attraction–learning duality enables reciprocity with theoretical neuroscience

- ***For agents as neurons (place cells), we consider that agent position is akin to place field location (e.g., place-field center-of-mass), inter-agent visibility to the presence of a synaptic connection, and inter-agent distance to the weight of that connection.***
- ***Phase-coupled attraction in the swarmalator formulation can thus be expressed as a form of Hebbian learning within a place-cell network.***

Connection; Weight difference — Phase coordination; Pre/post correlation — Short-term depression; Or, bounded synapse

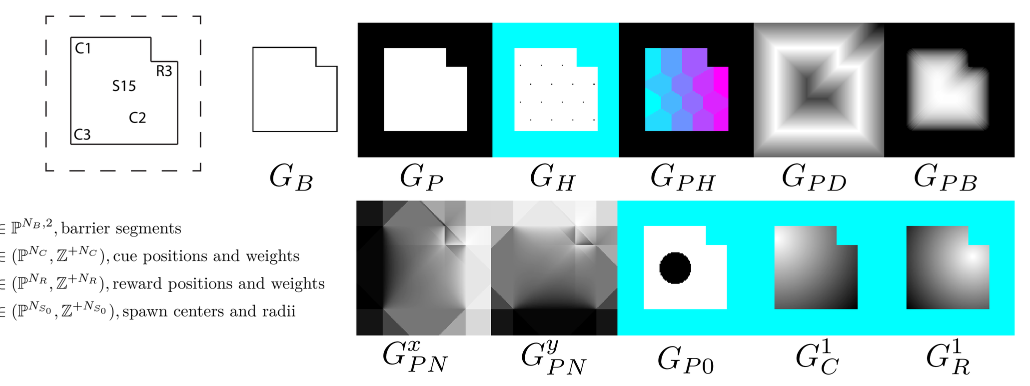
$$\dot{\mathbf{x}}_i = \frac{1}{N}\left[\sum_{j\notin i}^{N} \underbrace{\frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|}}_{\text{Connection; Weight difference}}\left(A + \underbrace{J\cos(\theta_j - \theta_i)}_{\text{Phase coordination; Pre/post correlation}}\right) - B\underbrace{\frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|^2}}_{\text{Short-term depression; Or, bounded synapse}}\right]$$

Kinetic–geometric constraints for complex environments in swarm & neural network models

- ***Unified 2D simulation platform for swarm–network dual models in complex environments***

$$G = \{\alpha, B, C, R, S_0, H, G_B, G_P, G_H, G_{PH}, G_{PD}, G_{PB}, G_{PN}, G_{P0}, X_0, D_{PC}, D_{PR}, V_{HH}, V_{HC}, V_{HR}\}$$

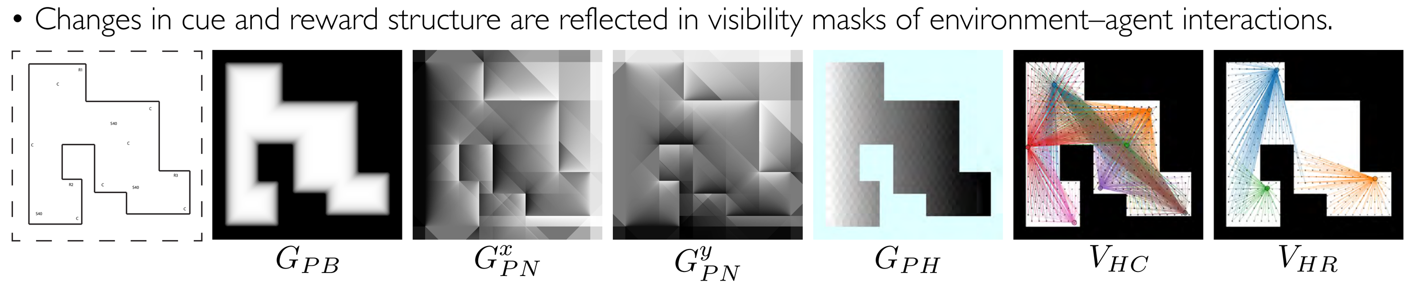
- Inputs can be provided by an easily designed vector image file in Tiny SVG format:



$\alpha :\in \mathbb{R}^1$, barrier spatial constant
$k_H :\in \mathbb{R}^1$, scale of hexagonal tiling grid
$w :\in \mathbb{Z}^{\geq 0}$, total width
$h :\in \mathbb{Z}^{\geq 0}$, total height
$\mathbb{P} := \mathbb{Z}^{0\dots w, 0\dots h}$, space of possible positions
$B :\in \mathbb{P}^{N_B, 2}$, barrier segments
$C :\in (\mathbb{P}^{N_C}, \mathbb{Z}^{+N_C})$, cue positions and weights
$R :\in (\mathbb{P}^{N_R}, \mathbb{Z}^{+N_R})$, reward positions and weights
$S_0 :\in (\mathbb{P}^{N_{S_0}}, \mathbb{Z}^{+N_{S_0}})$, spawn centers and radii

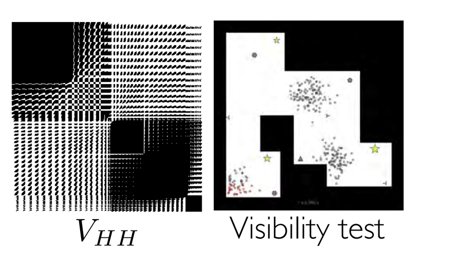
G_B G_P G_H G_{PH} G_{PD} G_{PB}
G_{PN}^x G_{PN}^y G_{P0} G_C^1 G_R^1

- Changes in cue and reward structure are reflected in visibility masks of environment–agent interactions.



G_{PB} G_{PN}^x G_{PN}^y G_{PH} V_{HC} V_{HR}

- Tile grid enables geometric masking of inter-agent (or recurrent neural) interactions based on current agent tile location
- Position updates are calculated according to swarmalator, neural network, or other model-based formulations using an internal, subjective, or 'preferred' position, $\boldsymbol{x_S}$
- Updates are maintained as vectors or averaged in single-agent paradigms, then velocities are filtered through momentum, kinetic energy constraints, and barrier-normal repulsion.

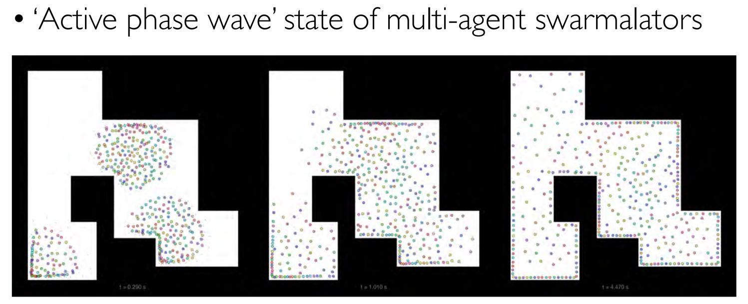


V_{HH} Visibility test

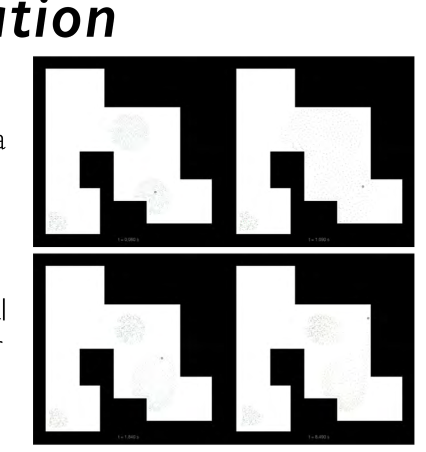
$$\Delta\boldsymbol{x_S} = F_{\text{swarmalator}}^{A^*, B^*, J, K}(\boldsymbol{x_S}, \boldsymbol{\theta})$$
$$\boldsymbol{v_S} := \Delta\boldsymbol{x_S}/\Delta t$$
$$\boldsymbol{v_S} := \langle\Delta\boldsymbol{x_S}\rangle_S/\Delta t, \text{for the single-agent models}$$
$$\boldsymbol{v_m} := \mu\boldsymbol{v} + (1-\mu)\boldsymbol{v_S}, \mu \sim 0.9$$
$$\boldsymbol{v_k} := \boldsymbol{v}_{\max}\tanh\left(\frac{|\boldsymbol{v_m}|}{\boldsymbol{v}_{\max}}\right)\frac{\boldsymbol{v}}{|\boldsymbol{v}|}$$
$$\hat{\boldsymbol{x}} := \boldsymbol{x} + \boldsymbol{v_k}\Delta t$$
$$b := G_{PB}(\hat{\boldsymbol{x}}), n_b := G_{PN}(\hat{\boldsymbol{x}})$$
$$\Delta\boldsymbol{x} = (1-b)\boldsymbol{v_k} - b|\boldsymbol{v_k}|n_b$$

Single-/multi-agent duality in simulation

- 'Active phase wave' state of multi-agent swarmalators



- Single-agent models in which a single 'physical' agent follows the resultant velocity flow of an internal 'mental' swarm of agents



Acknowledgements

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References

[1] S. Chung, A. A. Paranjape, P. Dames, S. Shen, and V. Kumar, "A Survey on Aerial Swarm Robotics," IEEE Trans. Robot., vol. 34, no. 4, pp. 837–855, 2018.
[2] J. D. Monaco, R. M. De Guzman, H. T. Blair, and K. Zhang, "Spatial synchronization codes from coupled rate-phase neurons," PLOS Comput. Biol., vol. 15, no. 1, pp. 1–42, 2019.
[3] K. P. O'Keeffe, H. Hong, and S. H. Strogatz, "Oscillators that sync and swarm," Nat. Commun., vol. 8, no. 1, p. 1504, 2017.
[4] A. Weinstein, A. Cho, G. Loianno, & V. Kumar. "Visual inertial odometry swarm: An autonomous swarm of vision-based quadrotors," IEEE Robotics and Automation Letters, 3(3), 1801–1807, 2018.
[5] J. Markdahl, J. Thunberg, and J. Goncalves, arXiv preprint arXiv:1807.10233, 2018.
[6] J. Markdahl, arXiv preprint arXiv:1808.00862, 2018.
[7] Y. Burak and I. R. Fiete, "Accurate path integration in continuous attractor network models of grid cells," PLOS Comput. Biol., vol. 5, e1000291, 2009.
[8] D. Bush and N. Burgess, "A hybrid oscillatory interference/continuous attractor network model of grid cell firing," Journal of Neuroscience 34, 5065 (2014).
[9] J. D. Monaco, J. J. Knierim, and K. Zhang. Sensory feedback, error correction, and remapping in a multiple oscillator model of place-cell activity. Front Comput Neurosci, 5:39, 2011.
[10] H. T. Blair, A. Welday, G. Shlifer, M. Bloom, and K. Zhang, "Stationary envelope synthesis (SES): a universal method for phase coding by neural oscillators," Front Neurosci Conf Abst: Comput Sys Neurosci 2010.
[11] V. G. Ivancevic and D. J. Reid, "Cognitive supervisor for an autonomous swarm of robots," Intelligent Control and Automation, 8(01):44, 2016.
[12] J. D. Monaco and L. F. Abbott, "Modular realignment of entorhinal grid cell activity as a basis for hippocampal remapping," J Neurosci, 31(25):9414–25, 2011.
[13] J. J. Knierim and K. Zhang, "Attractor dynamics of spatially correlated neural activity in the limbic system," Annu Rev Neurosci, 35:267–85, 2012.